

# The Geometry of Relativity

Richard A Jowsey

[richard@jowsey.org](mailto:richard@jowsey.org)

*“Imagination is more important than knowledge.  
For knowledge is limited to all we now know and understand,  
while imagination embraces the entire world.”  
— Albert Einstein*

## Abstract

A test particle of Planck mass is accelerated to the speed of light, in a 5+1D spacetime, where it has Planck momentum and Planck kinetic energy (in 3+1D spacetime, its kinetic energy goes to infinity). From the perspective of an inertial frame of reference, time becomes infinitely dilated in the test-particle's moving frame, and space becomes infinitely Lorentz-contracted. The test-particle's frame of reference is "4D-rotated" into mathematically imaginary dimensions, as the spatio-temporal phase angles<sup>[1]</sup> reach  $\pi/2$ . The Lorentz factor ( $\gamma$ ) is reformulated as a trigonometrical function of the gravito-electromagnetic phase angle ( $\phi$ ). Einstein's Special Theory of Relativity is shown to be a 4D approximation to relativity in 6D complex spacetime (i.e. in a quaternionic Kähler manifold).

## Introduction

In the late 19th century, Hendrik Lorentz, extending the theoretical insights of Sophus Lie (1871), Woldemar Voigt (1887), Oliver Heaviside (1889) and Joseph Larmor (1897), re-formulated James Clerk Maxwell's (1865) equations of electrodynamics to account for relativistic velocities.

In 1905, Henri Poincaré refined these equations into their modern form, referring to his formulation as the "Lorentz transformation", a subset of the Poincaré group of symmetry transformations. His expression for the Lorentz factor ( $\gamma$ ) maintains an invariant speed of light between two inertial frames of reference having a constant relative velocity between them:

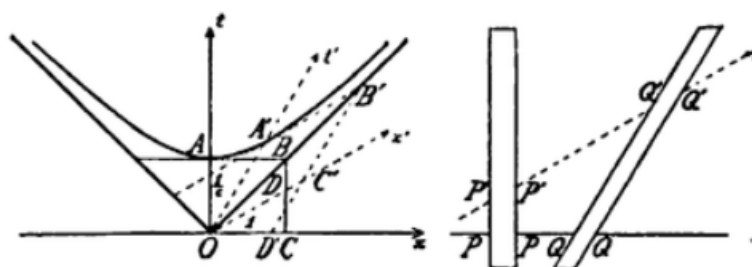
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

In his seminal paper *On the Electrodynamics of Moving Bodies*,<sup>[2]</sup> Albert Einstein (1905) showed that the laws of physics, notably the speed of light, are invariant in all inertial frames of reference. Einstein's special theory of relativity replaced the Galilean transformations of Newtonian physics with the Lorentz transformation.

Special relativity replaced the invariance of the time interval ( $dt$ ) between two events in three-dimensional space with an invariant hyperspatial interval ( $ds$ ) between two events in four-dimensional spacetime. Einstein conceptualised the relativistic spacetime transformation as a "mixing" of space and time, a 4D rotation of spatial length into time.

At velocities approaching the speed of light, in a tesseract spacetime geometry, a ruler became transformed into an imaginary clock. This counter-intuitive idea must have perplexed Einstein considerably, but his relativistic electrodynamics math worked perfectly (and  $E = mc^2$  fell out).

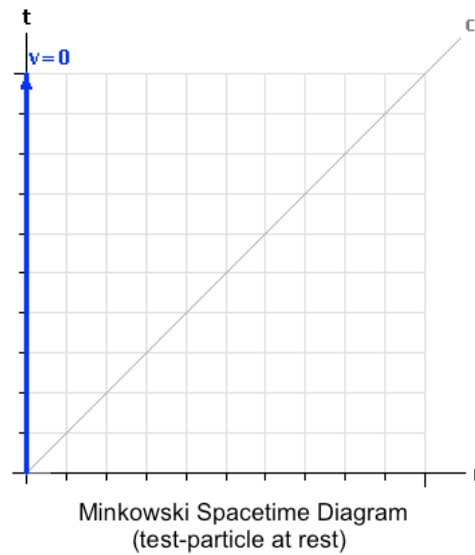
In 1908, the mathematician Hermann Minkowski reformulated Einstein's unified spacetime into a four-dimensional "Minkowski Space", wherein time ( $ct$ ) was simply another spatial dimension of the 4D geometry. Time, dimensioned in light-seconds, became an imaginary proxy for an axial distance.



Minkowski's Spacetime Diagram (1908)

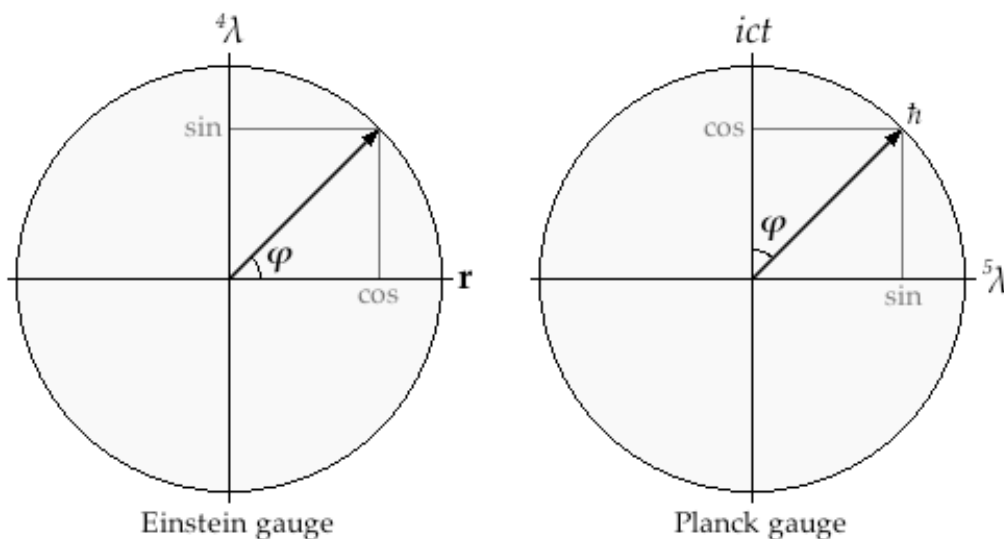
## Discussion

A Minkowski diagram plots the worldline of a test-particle in a single radial dimension of space ( $\mathbf{r}$ ), and through time ( $t$ ). The worldline of a particle having some constant velocity ( $v = \mathbf{r}/t$ ) illustrates the Lorentz transformation, or 4D rotation, of the frame of reference co-moving with the particle, and the time dilation and length contraction observable in the moving frame.

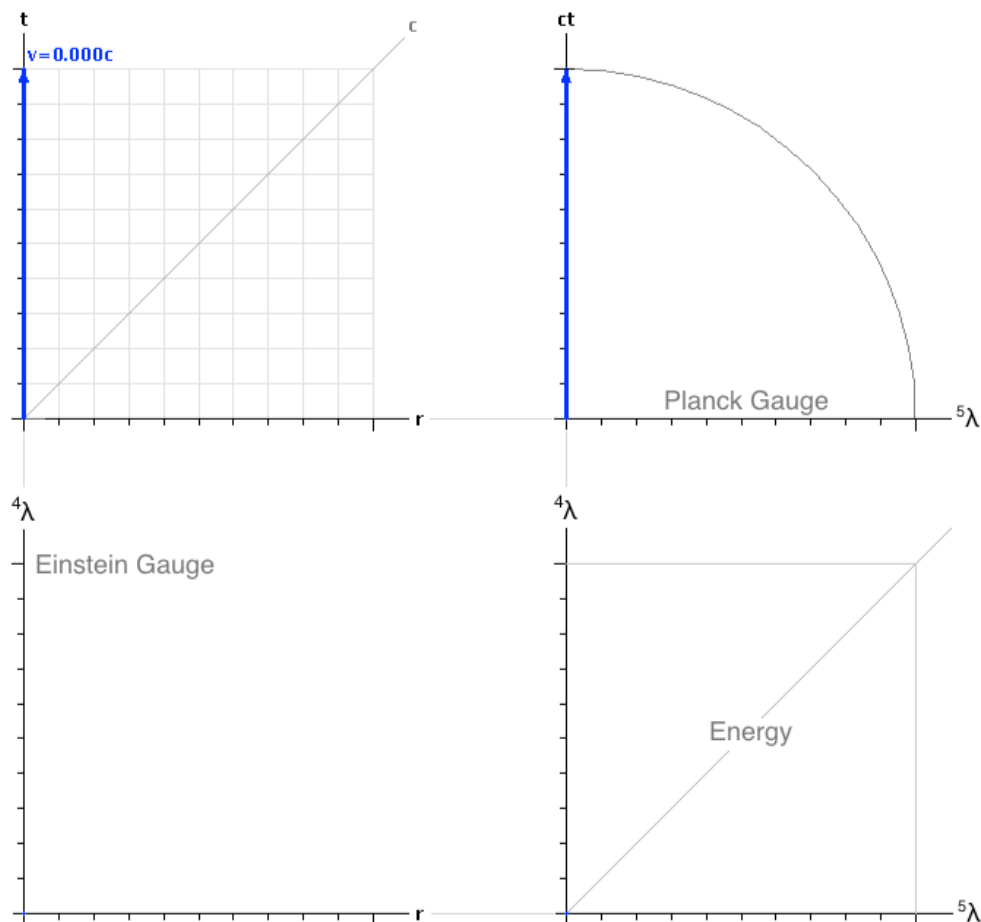


Deep dimensional analysis of the Planck units<sup>[1]</sup> reveals two extra spatial dimensions, which are intangible due to their being mathematically imaginary, i.e. they are Wick-rotated from, therefore orthogonal to, the three real spatial dimensions. They are unobservable, invisible, hidden dimensions of spacetime. They exist, mathematically, but they are not *real*.

The simplest model for such extra dimensions is provided by the complex plane, a geometric representation of complex numbers bounded by a space/time axis orthogonal to an imaginary axis. Within this complex plane, the unit vector sweeps out a circle in accordance with an exponential function of the form  $e^{i\varphi} = \cos \varphi + i \sin \varphi$ , having a radial modulus (absolute value) equal to Planck's reduced constant  $\hbar$ .



A six-dimensional extension of the Minkowski diagram requires the use of projective geometry. Each of the two imaginary spatial dimensions is projected against the 4D spacetime diagram, orthogonally. The position-momentum projection represents the  ${}^4\lambda$  Einstein gauge, and the time-energy projection represents the  ${}^5\lambda$  Planck gauge.<sup>[1]</sup>



The third projection (bottom-right) illustrates the  ${}^4\lambda \otimes {}^5\lambda$  cross-product intersection of the two imaginary spatial dimensions, and represents the totality of the particle's spatial-kinetic ( $pc$ ) and temporal-potential ( $mc^2$ ) energies.

The units of all the spatial dimensions (real and imaginary) are the Planck length, as is the unit of the "time" ( $ct$ ) axis, which represents the time required for light to propagate one Planck length in a vacuum.

This 6D geometry provides a mathematical framework for simulating a test-particle of Planck mass being accelerated to light-speed. The simulation enables verifying that time dilation and Lorentz length contraction occur precisely as predicted by 4D special relativity, ensuring quantitative equivalence with the special theory of relativity in flat Minkowski spacetime.

A particle of unit Planck mass  $m_P$  has the  $mc^2$  rest-mass energy equivalent of unit Planck energy ( $E_P$ ). Since it is at rest within an inertial frame of reference, its worldline is vertical on the Minkowski diagram, since its position is unchanging. It has zero momentum, and zero kinetic energy.

The ratio of the test-particle's velocity to light speed is  $\beta \equiv v/c = \sin(\varphi)$ , so the 6D Lorentz factor  $\gamma = 1/\sqrt{1 - \sin^2\varphi}$  is the inverse-cosine of the gravito-electromagnetic phase angle:

$$\varphi = \sin^{-1}(v/c) \qquad \gamma = \frac{1}{\cos \varphi}$$

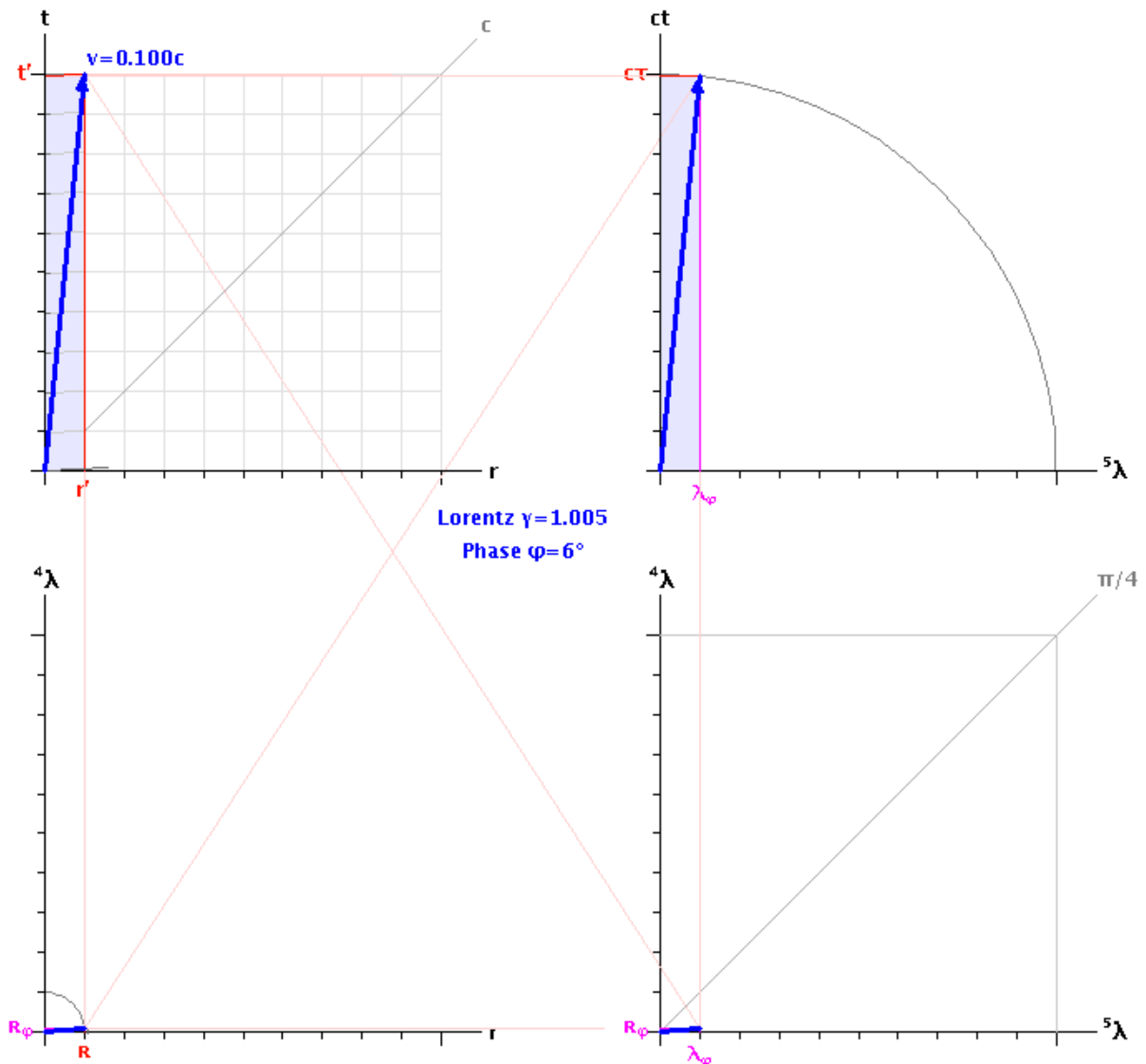
# GedankenExperiment

## Animated 6D simulation of a test-particle accelerating to light-speed<sup>[3]</sup>

At a velocity of  $0.1c$  (10% of light-speed), a test particle inhabits a slightly Lorentz-rotated frame of reference. The frame's gravito-electromagnetic phase angle  $\varphi = \sin^{-1}(v/c) = 0.100$  radians, signifying that the moving frame's spacetime is 6D-rotated by  $5.74^\circ$ .

The "six-dimensional curvature" of the moving frame of reference can be defined as  $\chi \equiv \sqrt{(2\varphi/\pi)}$ , where unit curvature represents the maximal 4D spacetime curvature, i.e. that of a frame of reference moving at light-speed (when phase  $\varphi$  reaches  $\pi/2$ ). At  $0.1c$ , the moving frame's 6D curvature is  $0.252$ .

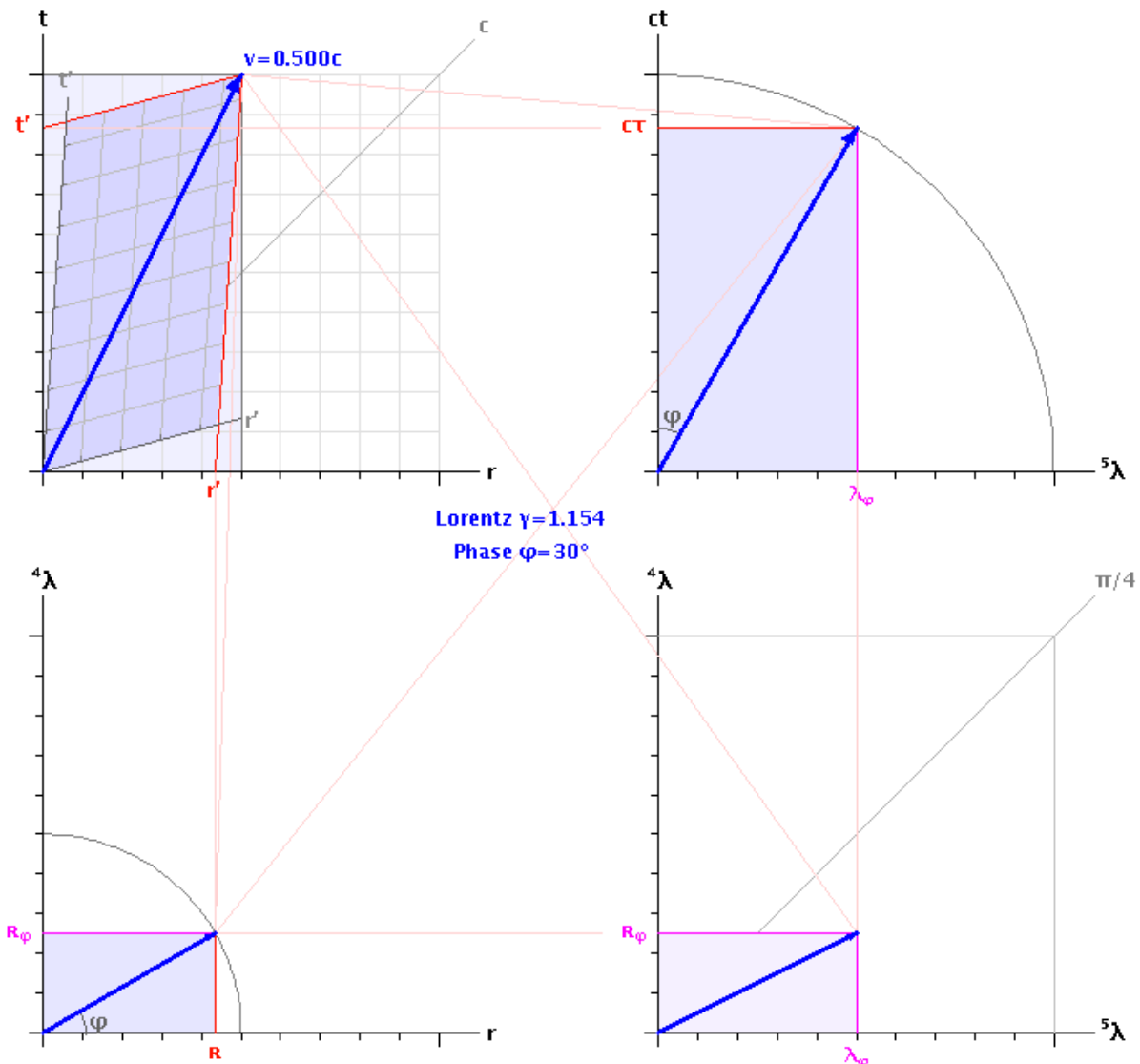
The Lorentz factor is formulated as  $\gamma = 1/\cos(\varphi)$  in six-dimensional spacetime, so time dilation simplifies to  $\Delta t' = \Delta t/\cos(\varphi)$ , and length contraction resolves to  $\Delta \mathbf{r}' = \Delta \mathbf{r} \cdot \cos(\varphi)$ . The particle's total energy is also a function of the phase angle, viz.  $E_t = \sqrt{((t \cdot \sin(\varphi))^2 + (\mathbf{r} \cdot \sin(\varphi))^2)}$ .



Since  $\cos(\varphi) = 0.995$ , a co-moving clock ticks at 1.005 seconds/second, and distance in the direction of motion is Lorentz-contracted by 0.995. The test-particle has acquired 0.1 units of Planck momentum (0.653 Ns), so has 6D kinetic energy of 0.1 Planck energy units ( $1.96 \times 10^8$  J).

Phase	Space (r)	Time (ct)	${}^4\lambda$	${}^5\lambda$	${}^4\lambda \otimes {}^5\lambda$	$\gamma mcv$
$5.739^\circ$	0.995	0.995	0.010	0.100	0.100	0.100

When the test-particle's velocity reaches  $0.5c$ , the gravito-electromagnetic phase angle  $\phi$  of its frame of reference is  $0.524$  radians. The moving frame's spacetime is now 4D-rotated by  $30^\circ$  ( $\pi/6$ ). The six-dimensional curvature  $\chi = 0.578$ .



Since  $\cos(\phi) = 0.866$ , a co-moving clock ticks at  $1.154$  seconds/second, and distance in the direction of motion is Lorentz-contracted by  $0.866$ . The test-particle has acquired  $0.5$  units of Planck momentum ( $3.262$  Ns), so possesses kinetic energy of  $0.5$  Planck energy units ( $9.786 \times 10^8$  J). The apparent (4D) relativistic kinetic energy  ${}^4E_k = mvc/\cos(\phi)$  is  $0.577$  units of Planck energy.

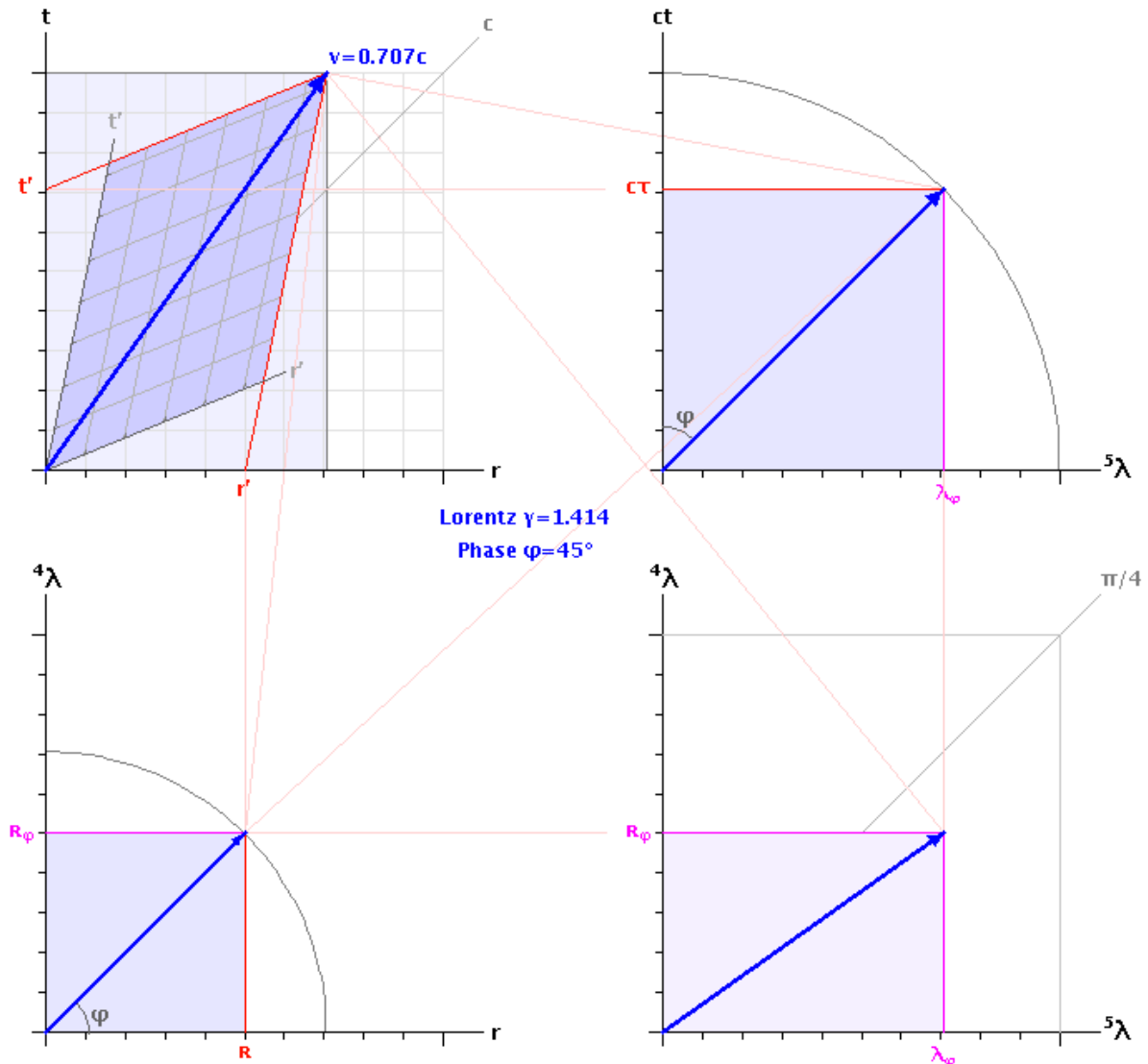
The bottom-left projection illustrates the imaginary plane which couples  $x,y,z$  space to the “Einstein gauge” spatial dimension,  ${}^4\lambda = m/c^2 = p/c^3 = E_k/c^4$ . The top-right projection shows the imaginary plane which couples (imaginary) time to the “Planck gauge” imaginary dimension,  ${}^5\lambda = \hbar/mc$ .

It is clearly apparent from the Minkowski diagram that the moving frame of reference is twisting around the velocity vector, rotating into mathematically imaginary dimensions of 6D spacetime, as the test-particle is accelerated towards light-speed. Time is dilating, so a clock in the moving frame is ticking slower. Space is contracting, so a co-moving ruler appears to be  $86.6\%$  shorter.

Phase	Space (r)	Time (ct)	${}^4\lambda$	${}^5\lambda$	${}^4\lambda \otimes {}^5\lambda$	$\gamma mcv$
$30^\circ$	$0.866$	$0.866$	$0.250$	$0.500$	$0.559$	$0.577$

This *gedanken-experiment* simulation presumes the test-particle has been dropped into an intense gravitational field, e.g. it's falling into a black hole. The  ${}^4\lambda$  and  ${}^5\lambda$  projections illustrate the temporo-spatial phase angles of the moving particle's frame of reference, relative to the phase angles of the lab frame, which are presumed to be zero.

When the particle's velocity reaches  $0.707c$ , i.e. 71% of light-speed, the gravito-electromagnetic phase angle  $\varphi$  of its frame of reference is 0.785 radians. The moving frame's spacetime is now 4D-rotated by  $45^\circ$  ( $\pi/4$ ). The six-dimensional curvature  $\chi = 0.707$ .



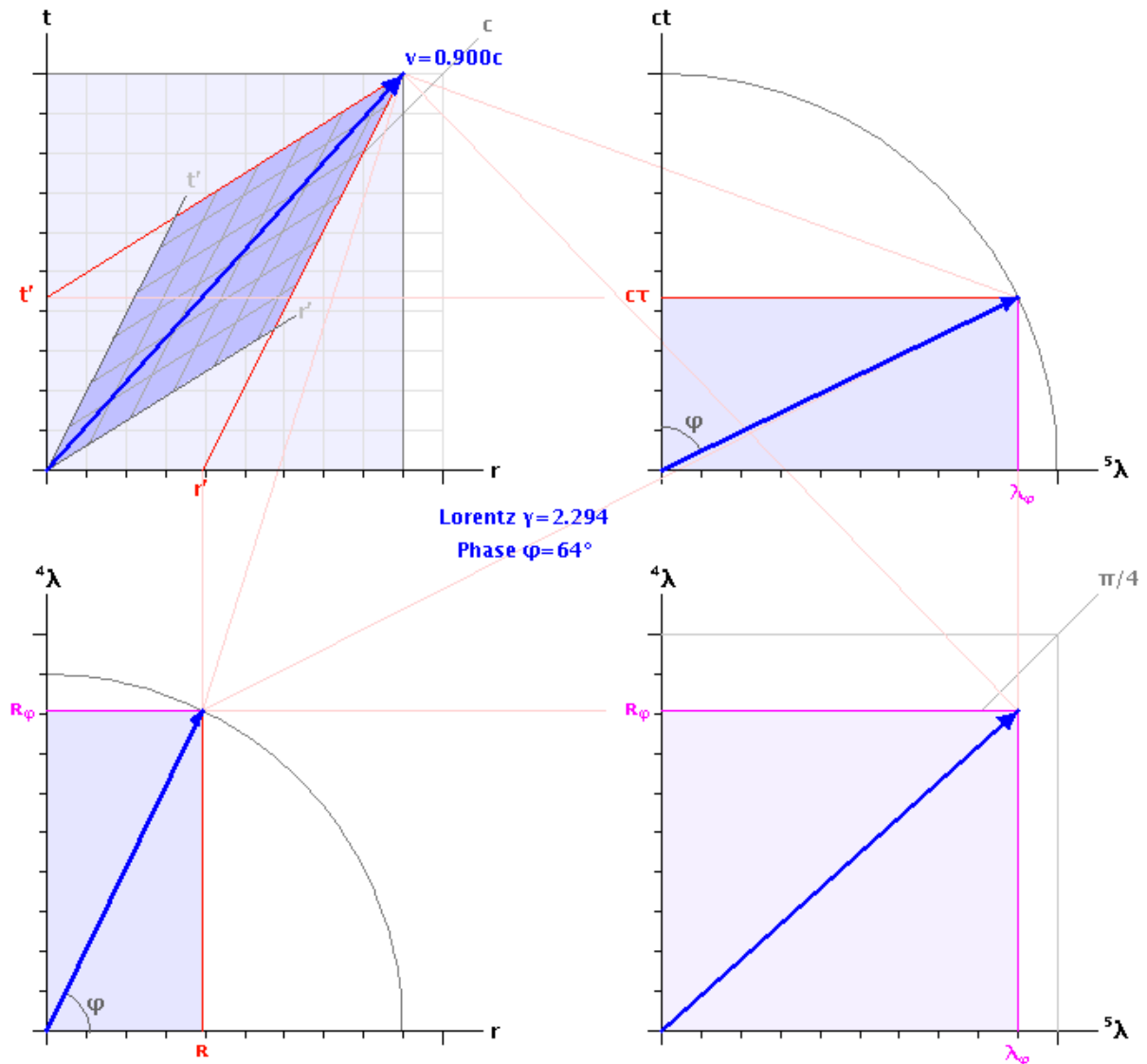
Since  $\cos(\varphi) = 0.707$ , a co-moving clock ticks at 1.414 seconds/second, and distance in the direction of motion is Lorentz-contracted by 0.707. The test-particle has acquired 0.707 units of Planck momentum (4.613 Ns), so possesses 6D kinetic energy of 0.707 Planck energy ( $1.384 \times 10^9$  J).

The length of the  ${}^4\lambda \otimes {}^5\lambda$  cross-product vector in the bottom-right projection is proportional to the total energy of the test-particle, per  $E_t = \sqrt{((r \cdot \sin(\varphi))^2 + (t \cdot \sin(\varphi))^2)}$ . When the phase angle reaches  $\pi/4$ , the 6D cross-product vector reaches 0.866 Planck energy units.

However, in four-dimensional Minkowski space, when  $\varphi$  reaches  $\pi/4$ , the 4D relativistic kinetic energy  ${}^4D E_k = mvc/\cos(\varphi) = 1$ , i.e. the particle's apparent kinetic energy reaches one Planck unit.

Phase	Space (r)	Time (ct)	${}^4\lambda$	${}^5\lambda$	${}^4\lambda \otimes {}^5\lambda$	$\gamma mcv$
$45^\circ$	0.707	0.707	0.500	0.707	0.866	1.000

When the particle's velocity reaches  $0.9c$ , the gravito-electromagnetic phase angle  $\varphi$  of its frame of reference is 1.119 radians. The moving frame's spacetime is now 4D-rotated by  $64^\circ$ . The six-dimensional curvature  $\chi = 0.844$ .

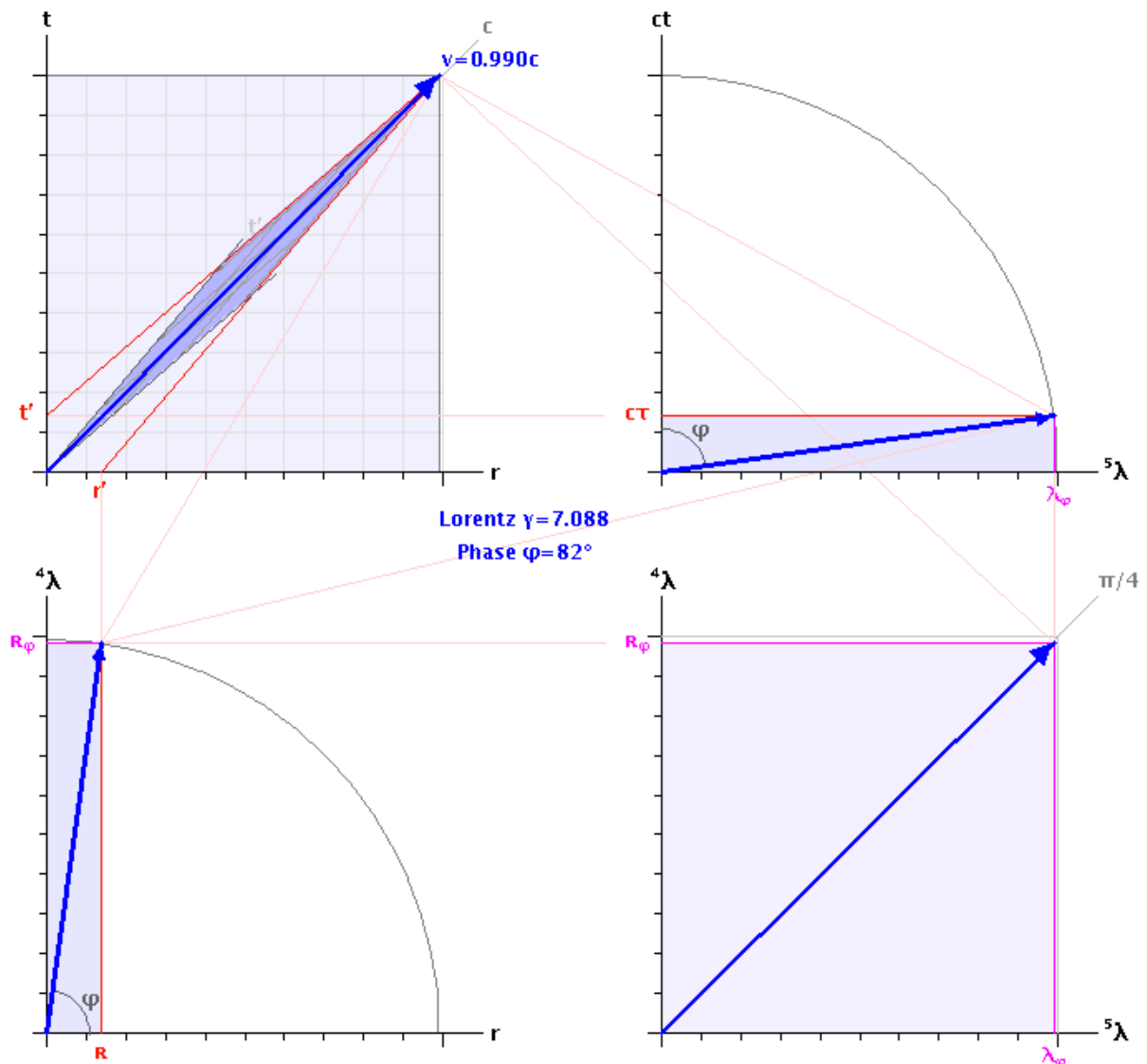


Since  $\cos(\varphi) = 0.436$ , a co-moving clock ticks at 2.294 seconds/second, and distance in the direction of motion is Lorentz-contracted by 0.436. The test-particle has acquired 0.9 units of Planck momentum (5.872 Ns), and therefore has 6D kinetic energy of 0.9 Planck energy ( $1.762 \times 10^9$  J). Its four-dimensional relativistic kinetic energy  ${}^{4D}E_k = mc/\cos(\varphi) = 2.065$  Planck energy units.

Phase	Space (r)	Time (ct)	${}^4\lambda$	${}^5\lambda$	${}^4\lambda \otimes {}^5\lambda$	$\gamma mcv$
64.158°	0.436	0.436	0.810	0.900	1.211	2.065



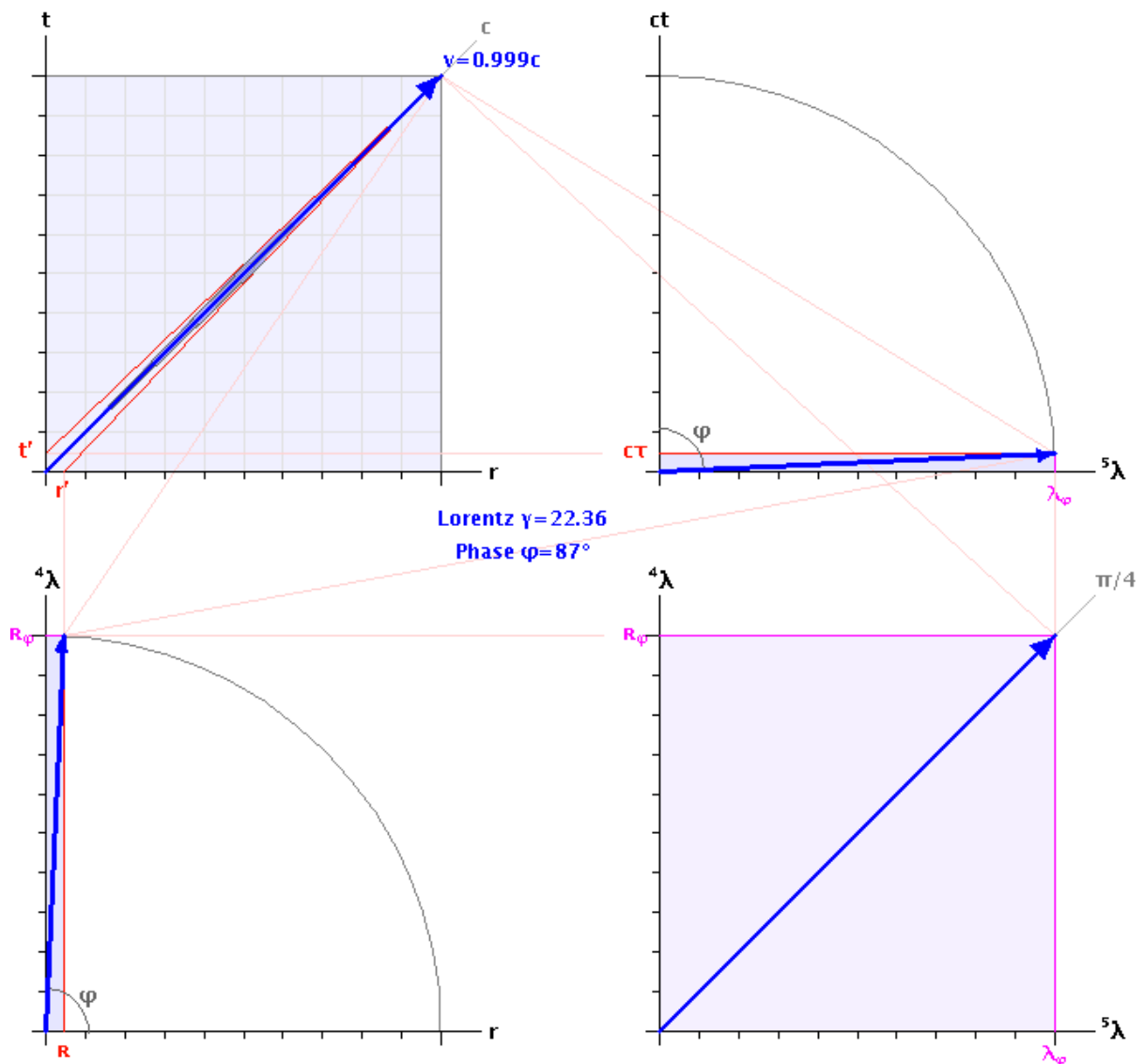
When the particle's velocity reaches  $0.99c$ , the gravito-electromagnetic phase angle  $\varphi$  of its frame of reference is 1.429 radians. The moving frame's spacetime is now 4D-rotated by  $82^\circ$ . The six-dimensional curvature  $\chi = 0.954$ .



Since  $\cos(\varphi) = 0.141$ , a co-moving clock ticks at 7.089 seconds/second, and distance in the direction of motion is Lorentz-contracted by 0.141. The test-particle has acquired 0.99 units of Planck momentum (6.459 Ns), so possesses 6D kinetic energy of 0.99 Planck energy ( $1.938 \times 10^9$  J). Its four-dimensional relativistic kinetic energy  ${}^4D E_k = mvc/\cos(\varphi) = 7.018$  units of Planck energy.

Phase	Space (r)	Time (ct)	${}^4\lambda$	${}^5\lambda$	${}^4\lambda \otimes {}^5\lambda$	$\gamma mcv$
$81.89^\circ$	0.141	0.141	0.980	0.990	1.393	7.018

When the particle's velocity reaches  $0.999c$ , the gravito-electromagnetic phase angle  $\varphi$  of its frame of reference is 1.526 radians. The moving frame's spacetime is now 4D-rotated by  $87^\circ$ . The six-dimensional curvature  $\chi = 0.986$ .

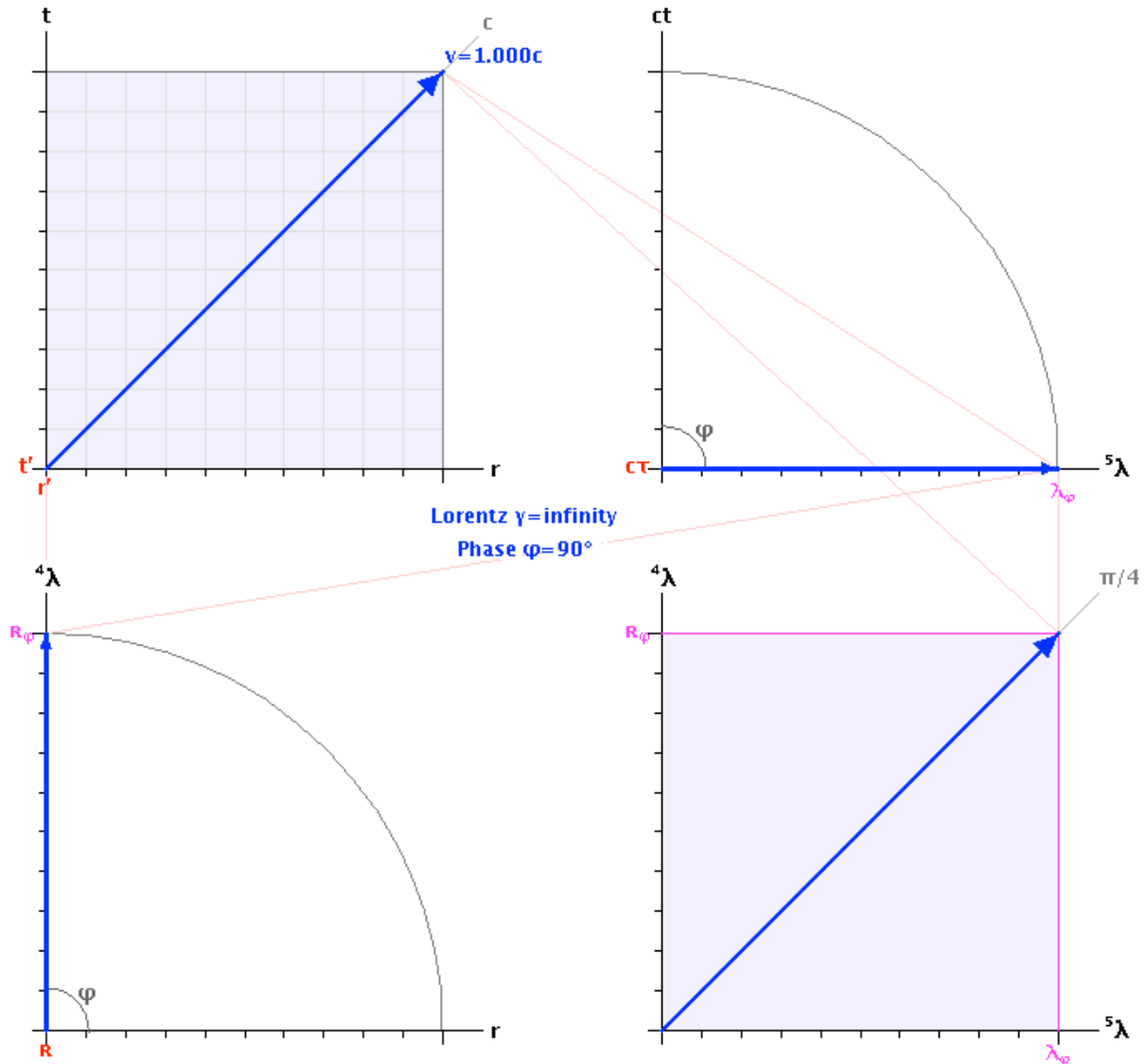


Since  $\cos(\varphi) = 0.045$ , a co-moving clock ticks at 22.37 seconds/second, and distance in the direction of motion is Lorentz-contracted by 0.045. The test-particle has acquired 0.999 units of Planck momentum (6.518 Ns), so has 6D kinetic energy of 0.999 Planck energy ( $1.954 \times 10^9$  J). Its four-dimensional relativistic kinetic energy  ${}^{4D}E_k = mvc/\cos(\varphi) = 22.34$  Planck energy units.

Phase	Space (r)	Time (ct)	${}^4\lambda$	${}^5\lambda$	${}^4\lambda \otimes {}^5\lambda$	$\gamma mcv$
$87.44^\circ$	0.045	0.045	0.999	0.999	1.413	22.34

At the speed of light, the gravito-electromagnetic phase angle  $\varphi$  of the particle's frame of reference is 1.57 radians. The moving frame's spacetime is now 4D-rotated by  $90^\circ$  ( $\pi/2$ ). The six-dimensional curvature  $\chi = 1.000$ , i.e. unit curvature.

Since  $\cos(\varphi) = 0$ , a clock co-moving with the fully-rotated frame stops ticking (time is infinitely dilated). Distance in the direction of motion is completely Lorentz-contracted, so the test particle appears to be infinitely thin in its direction of motion, as observed from the laboratory's inertial rest-frame.



It can be shown that Planck force ( $1.21 \times 10^{44}$  N) is required to accelerate Planck mass ( $2.176 \times 10^{-8}$  kg) to light-speed  $c$ , at which velocity it has acquired Planck momentum (6.525 Ns), and possesses 6D kinetic energy of one unit of Planck energy ( $1.956 \times 10^9$  J). At the speed of light, the test-particle's four-dimensional relativistic kinetic energy  ${}^4D E_k = \gamma m c^2 = E_P / \cos(\varphi)$  goes to infinity as the moving frame's phase angle reaches  $\pi/2$ .

Phase	Space (r)	Time (ct)	${}^4\lambda$	${}^5\lambda$	${}^4\lambda \otimes {}^5\lambda$	$\gamma m c v$
$90^\circ$	0	0	1.00	1.00	1.414	$\infty$

## Special relativity

As a massive test particle is accelerated in the 6D relativity simulator, the Lorentz transformation can be seen to rotate the moving frame of reference until it's orthogonal to the 4D spacetime plane of the rest-frame. The particle's spacetime frame appears to twist, then completely *vanish* into imaginary dimensions at light-speed (as if it were a photon). This hyperspatial vanishing act is a consequence of 4D Minkowski spacetime actually being embedded within a 6D complex manifold, a hyper-spacetime.

As it accelerates to relativistic velocities, the particle's rest-mass energy ( $mc^2$ ) and its relativistic kinetic energy of motion ( $\gamma pc$ ) can be seen to increasingly inhabit two imaginary spatial dimensions,  ${}^4\lambda$  and  ${}^5\lambda$ . At light-speed, the test-particle has zero coupling to real 3D space (a co-moving ruler has shrunk to its vanishing point). From the lab frame, the particle appears frozen in time, as if it had fallen onto a black hole's event horizon (a co-moving clock has stopped).

Within its proper spacetime frame, i.e. for a co-moving observer, the particle's matter-wave has a de Broglie wavelength of Planck length  $\ell_P$ , and is oscillating at Planck frequency  $f_P$ . However, from an inertial spacetime, the lab frame, the particle's wavelength appears infinite and its wave-packet's frequency appears to reach zero Hz. It fades to black.

The  ${}^4\lambda$  Einstein gauge can be formulated as proportional to momentum, viz.  ${}^4\lambda = p/c^3$ , or as proportional to kinetic energy, viz.  ${}^4\lambda = E_k/c^4$ .<sup>[1]</sup> At light-speed, spatial length contracts to zero in the direction of motion, and the particle's relativistic momentum ( $\mathbf{p} = \gamma m_{PC}$ ) reaches unit Planck length on the  ${}^4\lambda$  axis. As the phase angle  $\phi$  reaches  $\pi/2$ , the Lorentz factor  $\gamma = 1/\cos(\phi)$  goes to infinity, as the particle's kinetic momentum becomes mathematically imaginary.

The  ${}^5\lambda$  Planck gauge is canonically formulated as  ${}^5\lambda = \hbar/mc$ , the Compton wavelength, but it can also be usefully expressed as being inversely proportional to potential energy,  ${}^5\lambda = \hbar c/U_P$  (where  $U_P = m_{PC}c^2$ ). The particle's rest-mass energy ( $mc^2$ ) can be conceptualised as its "temporal momentum", i.e. the innate energy content of a mass as it flows through time (at *time-speed*), analogous to the kinetic momentum of the mass as it moves through space.

At light-speed, when time is infinitely dilated, the  ${}^5\lambda$  Planck gauge vector reaches unitary value, i.e.  ${}^5\lambda = \ell_P$ , one Planck length. A particle at rest is moving in time, with "temporal momentum" of  $mc$  and "temporal energy" of  $mc^2$ . When the particle has light-speed velocity in 6D space, its  $\sqrt{-1}$  momentum is  $imc$  and its kinetic energy is  $imc^2$ . The 4D Lorentz factor goes to infinity, but in flat 6D spacetime the Lorentz factor is a trigonometrical function of the angle of 6D rotation within a hyper-spherical geometry.

A relevant analogy is the annihilation of rest-mass when two antiparticles collide; instantaneously, their  $mc^2$  rest-mass energy (having "temporal momentum") is converted into radiant energy. These photons propagate in space at light speed, each having kinetic momentum  $hf/c$ . Photons have zero "temporal momentum", because their co-moving clocks have stopped.

In four dimensions, the Lorentz factor goes to infinity, but in 6D hyperspace it's simply reaching the zero point of an inverse-cosine function, as the moving frame's phase angle  $\phi$  reaches  $\pi/2$ . From a 4D perspective, the  ${}^4\lambda$  and  ${}^5\lambda$  imaginary spatial dimensions appear infinite in extent, because they are circular. In 6D projective geometry, infinity falls out of the mathematics at the north and south poles of a spherical hyper-surface. Spacetime goes exponential at its inflexion points.

Einstein's metaphorical "mixing" of time and space during a Lorentz transformation can be understood as a four-dimensional illusion, a *trompe-l'œil* "curvature" at the conformal boundary between 3/4D Minkowski spacetime and 5/6D hyper-spacetime. Space and time don't "mix" in 6D.

The  ${}^4\lambda \otimes {}^5\lambda$  bottom-right projection shows an imaginary plane coupling the Planck gauge (inverse-potential energy) with the Einstein gauge (kinetic energy). The magnitude of the resultant vector, the particle's total energy, is given by  $E_t = \sqrt{((\mathbf{r} \cdot \sin(\phi))^2 + (t \cdot \sin(\phi))^2)}$ , or alternatively by  $E_t = \sqrt{((p/c^3)^2 + (\hbar/mc)^2)}$ .

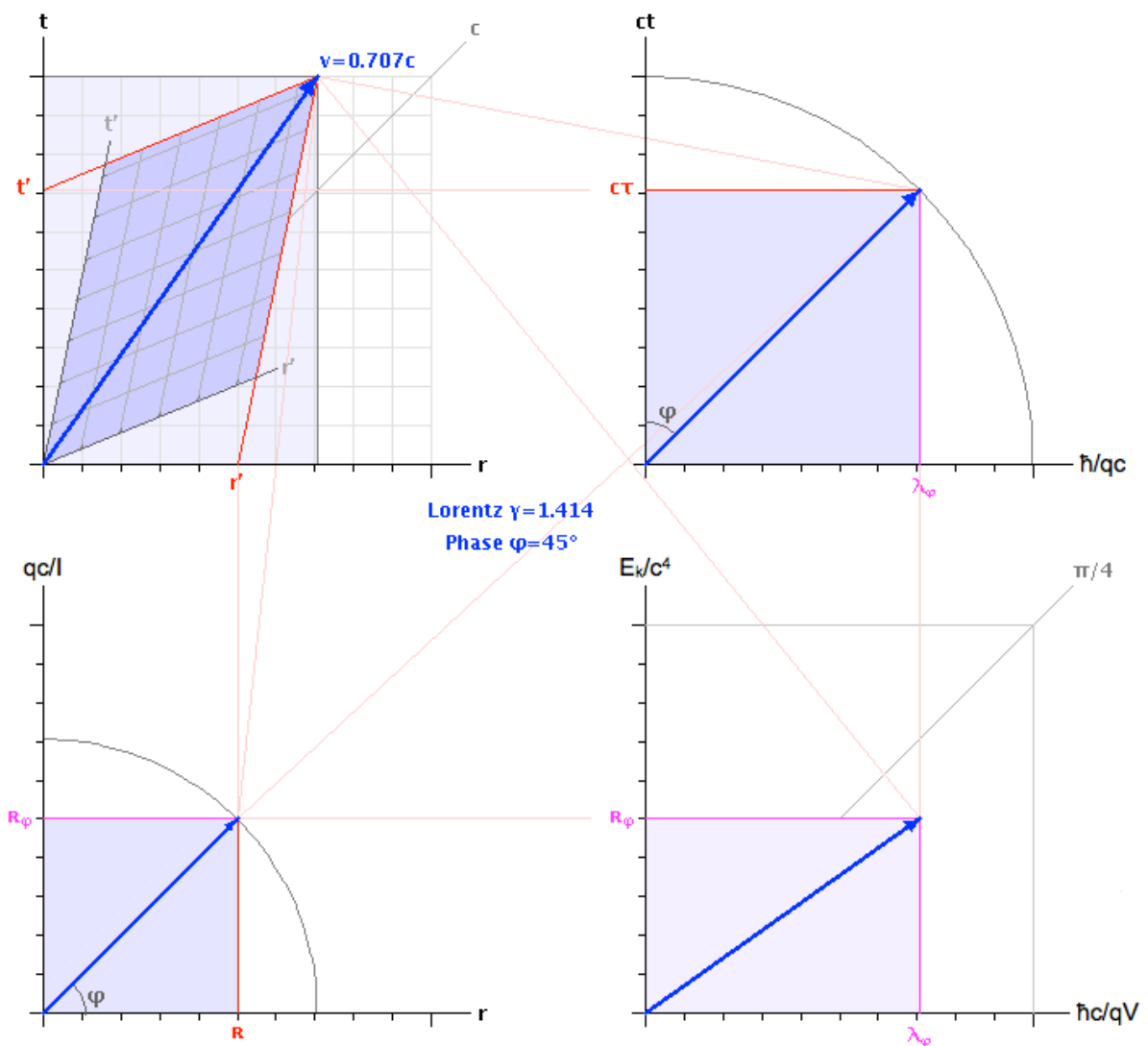
In a 6D vacuum, a Planck mass  $m_P$  at light-speed velocity has (imaginary) rest-mass energy of Planck energy,  $E_P = m_P c^2 \cdot \sqrt{-1}$ . Orthogonally, it has imaginary kinetic energy of Planck energy,  $E_k = m_P c^2 \cdot \sqrt{-1}$ . Therefore, in the six-dimensional frame of reference the particle's total energy is  $\sqrt{2}$  Planck energy. In the 3/4D lab frame, an observer perceives the particle's total energy at relativistic velocities as approaching infinity, due to the extreme time dilation and length contraction as  $v$  approaches  $c$ .

# Electromagnetism

The corresponding electrodynamic *gedanken-experiment* accelerates a hypothetical massless test-particle carrying the Planck unit of charge  $q_P$  ( $\sim 10^{-18}$  C) to light-speed  $c$ . In simulating six-dimensional relativistic electrodynamics, the imaginary spatial dimensions  ${}^4\lambda$  and  ${}^5\lambda$  are formulated in terms of charge  $q$ , electric current  $I$ , and voltage  $V$ .

The  ${}^4\lambda$  Einstein gauge can be expressed in terms of magnetic momentum, viz.  ${}^4\lambda = qc/I$ , or as a function of the kinetic energy of the magnetic field, viz.  ${}^4\lambda = E_k/c^4$ . The  ${}^5\lambda$  Planck gauge can be formulated as  ${}^5\lambda = \hbar/qc$ , or expressed as a function of potential energy  ${}^5\lambda = \hbar c/qV$ , i.e. this dimension is inversely proportional to potential energy  $U = qV$ .<sup>[1]</sup>

From the frame of reference of an inertial observer (the lab frame), a Lorentz transformation of the electric field associated with a moving charge results in the appearance of a magnetic field. Conversely, the magnetic potential induced into the space around a moving charge transforms into a purely electrostatic potential when observed from a co-moving frame of reference.



At relativistic velocities, the “mixing” of the magnetic and electric potentials in 4D Minkowski spacetime is revealed to be a 6D rotation of the moving frame of reference. The magnetic potential is associated with the  ${}^4\lambda$  Einstein gauge, while the  ${}^5\lambda$  Planck gauge represents the electric potential.

As the charged particle’s velocity increases and time dilates, the electrical energy becomes increasingly imaginary in the Planck gauge. Concomitantly, the magnetic energy stored in space around the speeding charge increases, and also becomes increasingly imaginary in the Einstein gauge.

Observed from the inertial lab frame, the magnetic momentum of a Planck-charged particle moving at light-speed is exactly equivalent to the relativistic momentum of a luminal-velocity Planck mass, i.e.  $\gamma q_P c = \gamma m_P c = \text{unit Planck momentum}$ .

The kinetic energy stored in the magnetic field surrounding a luminal-velocity Planck charge is equivalent to Planck energy,  ${}^{4D}E_k = \gamma q_P c^2$ , just as the kinetic energy stored in spacetime (as gravitational “curvature”) by a Planck mass with velocity  $c$  is given by  ${}^{4D}E_k = \gamma m_P c^2$ . Mathematically, an electromagnetic field is isomorphic to the “gravito-temporal” field, but these mass and charge fields are orthogonal in six-dimensional spacetime.

As is the case for a mass at rest, the “temporal momentum” associated with a charge at rest (i.e. moving in time) appears to diminish as time dilates, as its co-moving clock ticks more slowly. Since the Planck gauge is inversely-proportional to potential energy, as this imaginary  ${}^5\lambda$  quantity increases with higher velocity, the “temporal potential energy” is concomitantly diminishing. Thus an increase in the phase angle increases the potential energy stored in higher-dimensional angular momentum.

For a charged particle moving at light-speed, the kinetic and potential energy has been completely 6D-rotated into imaginary dimensions. As observed from the lab frame, a clock co-moving with the particle stops, and a co-moving ruler contracts to a vanishing point (leaving only its smile).<sup>[4]</sup>

## Conclusions

The core mathematical concept of Special Relativity is the Lorentz transformation, a subset of the Poincaré group of symmetry transformations, which Einstein conceptualised as the “mixing” of time and space in 4D Minkowski spacetime.

In six-dimensional complex spacetime, the Lorentz rotation is the 6D rotation of a gravito-electromagnetic phase angle  $\phi$  within two complex planes. Because the ratio of velocity to light-speed  $\beta = v/c$  becomes  $\sin(\phi)$ , the 6D Lorentz factor becomes  $1/\cos(\phi)$ .

There is no “mixing” of space and time in 6D. Spatial intervals are what rulers measure, and temporal intervals are what clocks measure. Rulers for measuring imaginary spatial dimensions are circular, with a circumference of Planck’s constant of action,  $h$ .

Einstein interpreted time dilation and length contraction as “spacetime curvature”. This concept eventually led him to a general theory of relativity pertaining to gravitation in curved 4D spacetime.

In six dimensional spacetime geometry, this curvature can be understood as an illusion at the “conformal boundary” between 4D and 6D spacetime, analogous to a 3D hologram at every point in space. We are only able to observe the 3D surface of higher-dimensional mathematical entities (n-dimensional spheres). N-dimensional phenomena are only comprehensible as a 3D hologram.

The hard vacuum of deep space doesn’t warp nor curve. This counter-intuitive idea is replaced by higher-dimensional spacetime “curvature”, i.e. rotation of n-dimensional vectors in hyperspatial circular dimensions.

Such spatial dimensions *exist* mathematically, and ontologically, but are not “real”. They can never be observed. Nor are they *knowable*, epistemologically, except through mathematical imagination.

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